Reducing the number of calibration patterns for the two-by-two dot centering model

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ABSTRACT

The two-by-two dot centering model enables predicting the spectral reflectance of color halftones and does not depend on a specific halftoning algorithm. It requires measuring the reflectances of a large number of two-by-two calibration tile patterns. Spectral measurement of hundreds or thousands of tile patterns is cumbersome and time consuming. In order to limit the number of measurements, we estimate the reflectances of a large majority of two-by-two calibration tile patterns from a small subset comprising less than 10% of all tile patterns. Using this subset of measured two-by-two calibration tile patterns, we perform a linear regression in the absorptance space and derive a transformation matrix converting tile pattern colorant surface coverages to absorptances. This transformation matrix enables calculating the absorptance of all remaining two-by-two tile patterns. For a cyan, magenta and yellow print, with 72 two-by-two measured calibration tile patterns, we are able to create a two-by-two dot centering model having an accuracy only slightly below the accuracy of the model with the fully measured set of 1072 two-by-two tile patterns.

Keywords: Spectral reflectance prediction, color halftone, two-by-two model, absorptance, Saunderson correction, linear regression

1. INTRODUCTION

The spectral characterization of a printer aims at relating the input control values of the printer to the spectral reflectance of the printed color. Most spectral prediction models are established by learning from a set of measurements called calibration set. Once calibrated, the models predict the reflectances of halftones printed with known nominal surface coverages of the inks.

The deposited ink dot surface coverage in a print is generally larger than the nominal surface coverage, yielding a "physical" dot gain responsible for the ink spreading phenomenon. In addition, the lateral scattering of light within the paper substrate and the internal reflections at the interface between the print and the air are responsible for what is generally called "optical" dot gain, also known as the Yule-Nielsen effect. The two-by-two dot centering model [1] accounts for both physical and optical dot gains considering the printer and the halftoning method as a black box. This model uses the measured reflectance of all different two-by-two pixel tile configurations. It is then able to predict the reflectance of any halftone by determining at each position the corresponding two-by-two tile patterns and by integrating their measured reflectances in the Yule-Nielsen formula. However, this halftone independent spectral prediction model has the drawback of requiring a very large number of measurements for characterizing a given 3 or 4-ink print setup.

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In this contribution, we propose to estimate the reflectances of most two-by-two tile patterns using only a small subset of measured two-by-two tile reflectances. Assuming the surface coverages of the two-by-two tiles as colorant concentrations, we build a linear relationship between surface coverages of the measured two-by-two tile patterns and their corresponding absorptances which are derived from the reflectances. Then, a transformation matrix mapping the colorant surface coverages to their absorptance spectra is computed by performing a linear regression. Feeding the surface coverages of non-measured two-by-two tile patterns to this transformation matrix, we predict their absorptances and compute their reflectances. Then, using the estimated reflectances, we predict the reflectances of color halftone patches using the normal two-by-two dot centering model.

To test the performance of the proposed method, we compare the accuracy of the predictions of a two-by-two dot centering model using a fully measured calibration set and the prediction accuracy using a set mainly composed of predicted tile pattern reflectances. For a cyan, magenta and yellow inkjet print at 600 dpi, with 72 two-by-two measured calibration tile patterns, we are able to create a two-by-two dot centering model having accuracy slightly below the accuracy of the model with the fully measured set of 1072 two-by-two tile patterns.

2. TWO-BY-TWO DOT CENTERING MODEL

More than ten years ago, Wang [1] proposed a halftone independent spectral prediction model. According to this model, one needs only seven independent calibration two-by-two tile patterns to characterize black and white halftones. Based on these seven calibration tile patterns, the two-by-two dot centering model is able to predict all possible configurations of black/white halftones. These seven calibration tile patterns with their symmetrical pairs along the horizontal and vertical axes describe all possible black/white distributions within a two-by-two pixel tile. Figure 1a shows the seven possible two-by-two calibration patterns G0 to G6. In order to create macro patches containing only a single pattern, these patterns are replicated and sent to the printer. Their reflectance spectra are measured with a spectrophotometer.

The reflectances of these seven tile patterns form the two-by-two calibration set. In order to predict the reflectance of a halftone, the corresponding halftone element is analyzed. At each location, the corresponding two-by-two tile pattern is found (Figure 1b). Then, the number of occurrences of each two-by-two calibration tile pattern within the halftone, including its symmetrical counterparts, is counted. After calculating the number of occurrences of each two-by-two calibration tile pattern, the halftone reflectance $R(\lambda)$ is predicted using the Yule-Nielsen modified spectral Neugebauer (YNSN):

$$R(\lambda) = \left(\frac{\sum\limits_{m=0}^{6} i_m R_m(\lambda)^{1/n}}{\sum\limits_{m=0}^{6} i_m} \right)^n \tag{1}$$

where i_m is the number of occurrences of the calibration tile pattern G_m, and R_m is its corresponding measured reflection spectrum. The *Yule-Nielsen n-value* accounts for the non-linear relationship between the reflectances of the calibration tile patterns and the global reflectance of the analyzed halftone pattern.

Extending the two-by-two model to color halftones requires a significant increase in the number of two-by-two calibration tile patterns. This is due to the large number of possible arrangements of different inked colorant pixels within a two-by-two pixel tile. Since there are 4 possibilities in a two-by-two tile, the number of possible colorant arrangements per two-by-two pixel tile is N^4, where N is the number of colorants. Colorants are formed by single inks, all possible superpositions of single inks and the paper white. For example, in the case of a CMY print (8 colorants), there are $8^4 = 4096$ possible color arrangements. After removing horizontally and vertically symmetric

two-by-two tiles, 1072 independent tile patterns remain. For N colorants, the total number of independent patterns $P(N)$ is given by Equation (2), see [2, pp. 470-479]

$$P(N) = \frac{N!}{(N-4)!4!}6 + \frac{N!}{(N-3)!3!}9 + \frac{N!}{(N-2)!2!}5 + \frac{N!}{(N-1)!} \qquad (2)$$

where for $N = 3$ and for $N = 2$ the first term and the first two terms are omitted, respectively. For black and white halftones there are $P(2) = 7$ independent tile patterns and for CMY halftones, there are $P(8) = 1072$ independent tile patterns. For a 4-ink print, the two-by-two dot centering model becomes intractable with $P(16) = 16576$ tile patterns. We therefore need a method for inferring the reflectances of a large number of tile patterns, given the measured reflectances of a small subset of them.
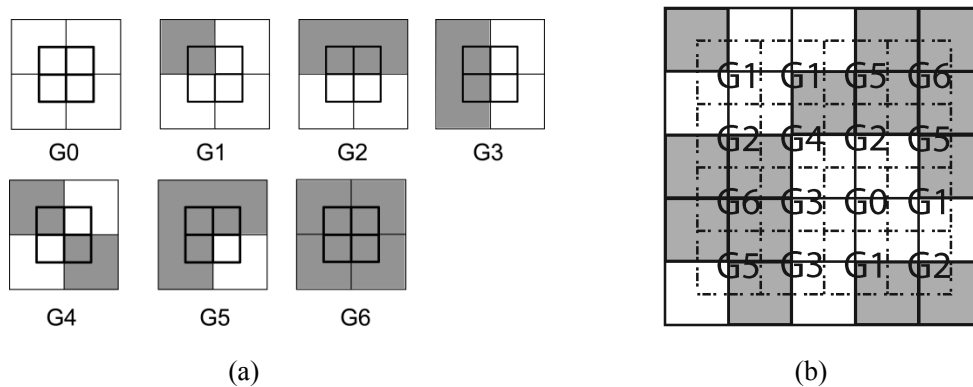

Figure 1. (a) The seven calibration patterns for the black and white two-by-two model. (b) Example of a halftone with the corresponding mapped two-by-two patterns.

3. TWO-BY-TWO PATTERN TILE REFLECTANCE PREDICTION RELYING ON SURFACE COVERAGES

We would like to estimate the spectral reflectance of unknown two-by-two tile patterns from known, measured tile patterns (Figure 2). We propose to estimate the reflectances of most two-by-two tile patterns relying only on a small subset of measured two-by-two tile reflectances. We assume that within the two-by-two tiles, surface coverages of colorants are equivalent to relative concentrations. We therefore work in the absorptance space and establish a linear relationship between surface coverages (0, 0.25, 0.5, 0.75, and 1) and two-by-two tile absorptances. We convert measured reflectances of the selected subset of two-by-two tile patterns into absorptances. By performing a linear regression in the absorptance space, a transformation matrix is established which maps the colorant surface coverages of two-by-two tiles to their absorptance spectra. This transformation matrix enables estimating the unknown absorptances and, consequently, the reflectances of those two-by-two tile patterns that have not been measured. Then, the two-by-two dot centering model can be used to predict the reflectances of color halftone patches.

Let us explain the algorithm in detail for a CMY print with 8 colorants and 1072 two-by-two calibration patterns. The first step is to choose a subset of two-by-two patterns and use it as the training dataset. There are several possibilities for selecting this subset. If we know the probability of occurrence of each calibration pattern for a given halftoning algorithm of known halftoning parameters, we can include the most probable two-by-two tile patterns in the training set and estimate all others. Another approach consists in selecting two-by-two tile patterns

which are well distributed across the printer gamut having different pattern orientations. In the present contribution, in order to apply our approach and verify its accuracy on a non-optimized set of learning patterns, we randomly select the two-by-two tile pattern training set.
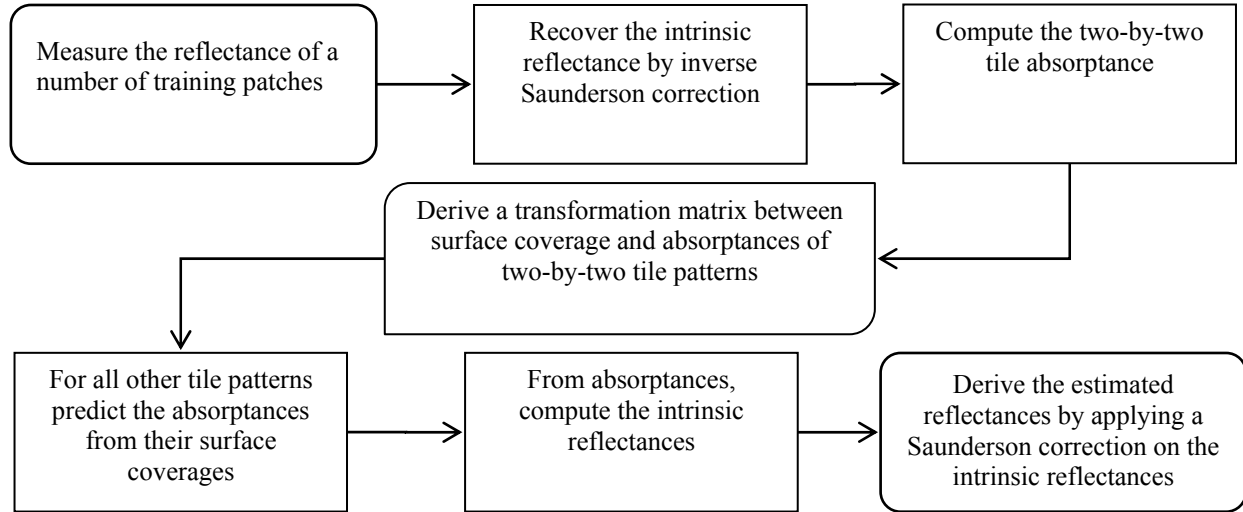

Figure 2. Predicting the reflectance of two-by-two calibration tile patterns by setting up a matrix establishing a relation between tile surface coverages and tile absorptances using a subset of tiles with known reflectances.

We also try to verify the impact of the size of the training set. When the number of measurements increases, we expect a decrease in the prediction error of the resulting two-by-two model.

The measured reflectance $R_m(\lambda)$ is composed of the intrinsic reflectance $\rho(\lambda)$, and of the Fresnel reflectances at the boundaries of the print-air interface. Since we are interested in the absorptance of the two-by-two tile patterns, we apply an inverse Saunderson correction on the measured reflectance spectra of the training set in order to first recover the intrinsic reflectances. The expression for the Saunderson correction is [3]

$$R_m(\lambda) = K \cdot r_s + \frac{(1-r_s)(1-r_i)\rho(\lambda)}{1-r_i \cdot \rho(\lambda)} \tag{3}$$

where r_s is the Fresnel specular reflection at the air-print interface, K the portion of specular reflection captured by the spectrophotometer and r_i the Fresnel reflection at the print-air interface, averaged over all incident angles. The overall Fresnel reflection at the print-air interface has been calculated by Judd [4] as $r_i = 0.6$ for an index of refraction of 1.5. For a given incident angle, r_s is constant and depends only on the refractive index of the print (at 45°, for a refractive index of 1.5 and $r_s = 0.04$). We obtain the intrinsic reflectance of the two-by-two pattern by applying the inverse Saunderson correction, derived from Equation (3)

$$\rho(\lambda) = \frac{R_m(\lambda) - K \cdot r_s}{1 + (1-K)r_i \cdot r_s + r_i \cdot R_m(\lambda) - r_i - r_s} \tag{4}$$

Within the narrow space of a two-by-two tile, we assume that surface coverages are equivalent to colorant *concentrations*. As proposed by Berns [5], we select the absorptance space as the space in which we establish a linear relationship between surface coverages and absorptances.

Let us now calculate the absorptance from the previously calculated intrinsic reflectance. If we assume that light travels twice through the halftone colorant layer, we can express the intrinsic two-by-two tile reflectance $\rho_i(\lambda)$ as

$$\rho_i(\lambda) = \rho_p(\lambda)T^2(\lambda) \tag{5}$$

where T^2 is the attenuation of light traversing twice the colorant halftone layer and ρ_p is the intrinsic reflectance of paper. The attenuation T of the colorant halftone layer can be expressed by the Beer-Bouguer law

$$T(\lambda) = e^{-\varepsilon(\lambda) \cdot c \cdot d} \tag{6}$$

where $\varepsilon(\lambda)$ is the molar extinction coefficient, c is the concentration and d is the thickness of the colorant halftone layer. Since the absorptance is defined as $K(\lambda) = \varepsilon(\lambda) \cdot c \cdot d$, we obtain from Equations (5) and (6)

$$\rho_i(\lambda) = \rho_p(\lambda)e^{-2K(\lambda)} \tag{7}$$

By inverting Equation (7), we obtain the spectral absorptance $K(\lambda)$

$$K(\lambda) = -\frac{1}{2}\ln\left(\frac{\rho_i(\lambda)}{\rho_p(\lambda)}\right) \tag{8}$$

Since for a CMY print we have 8 colorants, an 8 component vector characterizes the surface coverage of a two-by-two pattern. As an example, Figure 3 shows 2 two-by-two calibration tile patterns. The left hand side tile is composed of surface coverage of 25% cyan, 25% magenta, 25% yellow and 25% black and the right hand side is composed of surface coverages of 25% white, 25% cyan and 50 % blue. These surface coverages are expressed as vectors $\mathbf{a} = (0, 0.25, 0.25, 0.25, 0, 0, 0, 0.25)$ and $\mathbf{b} = (0.25, 0.25, 0, 0, 0.50, 0, 0, 0)$, respectively, where the order of colorants in the vector is white (w), cyan (c), magenta (m), yellow (y), blue (b), green (g), red (r), and black (k).

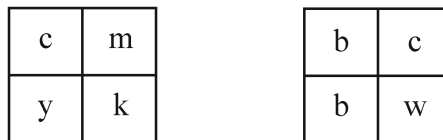| c	m
y	k

b	c
b	w

Figure 3. Two examples of two-by-two tiles.

The stated linear relationship in absorptance space implies that a given two-by-two tile absorptance is a linear combination of base tile absorptances:

$$\mathbf{k} = c_1\mathbf{k}_1 + c_2\mathbf{k}_2 + ... + c_8\mathbf{k}_8 \tag{9}$$

where $\mathbf{k}$ is a column vector expressing absorptances at visible wavelengths, e.g. 36 discrete wavelengths between 380nm and 730nm at 10nm intervals, where c_1 to c_8 are the surface coverages of the considered 8 colorants and $\mathbf{k_i}$ are the colorant absorptances derived from the training set. Equation (9) can be formulated in vector-matrix form

$$\begin{bmatrix} k_1 \\ k_2 \\ \vdots \\ k_{36} \end{bmatrix} = \begin{bmatrix} k_{1,1} & k_{1,2} & \cdots & k_{1,8} \\ k_{2,1} & k_{2,2} & \cdots & k_{2,8} \\ \vdots & \vdots & \vdots & \vdots \\ k_{36,1} & k_{36,2} & \cdots & k_{36,8} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ \vdots \\ c_8 \end{bmatrix} \tag{10}$$

or equivalently

$$\mathbf{k} = \mathbf{Mc} \tag{11}$$

where $\mathbf{c}$ is the surface coverage 8 component vector of a two-by-two pattern and $\mathbf{M}$ is the 36 by 8 matrix, composed of the colorant vectors $\mathbf{k_i}$. Matrix $\mathbf{M}$ gives the relationship between colorant surface coverages and spectral absorptance. We can derive the coefficients of matrix $\mathbf{M}$ with a set of training two-by-two tiles significantly larger than the number of unknowns. With a set of m training tile patterns with known surface coverage vectors $\mathbf{c_i}$ and absorptances $\mathbf{k_i}$, we obtain an overdetermined linear system

$$[\mathbf{k_1}, \mathbf{k_2}, ..., \mathbf{k_m}] = \mathbf{M}[\mathbf{c_1}, \mathbf{c_2}, ..., \mathbf{c_m}] \tag{12}$$

or in short form

$$\mathbf{K} = \mathbf{MC} \tag{13}$$

Matrix $\mathbf{K}$ is the *absorptivity* matrix with a size of 36 by m, $\mathbf{C}$ is an 8 by m matrix containing the colorant surface coverages and $\mathbf{M}$ is the 36 rows by 8 columns transformation matrix. We can solve this linear system by minimizing $\lVert \mathbf{K} - \mathbf{MC} \rVert$, see [6]. The corresponding MATLAB function *pinv* calculates the pseudo-inverse. Having obtained matrix $\mathbf{M}$, we can compute the absorptance of any two-by-two tile from its surface coverages by applying Equation (11).

After calculating the absorptance with the transformation matrix obtained by linear regression, we convert absorptances to intrinsic reflectances. This is accomplished by Equation (7). Finally, by applying the Saunderson correction given by Equation (3), we obtain the predicted reflectance on the air side of the two-by-two tile.

4. RESULTS

4.1 Two-by-two pattern estimation performance

The experiments were carried out on an inkjet printer (Canon Pixma Pro 9500 at 600 dpi) by printing color halftone test samples with mutually rotated classical clustered-dot halftones at three screen frequencies (75 lpi, 100 lpi and 125 lpi) and with blue noise diffused dither halftones. The CMY test prints were printed on Canon MP-101 paper with all combinations of surface coverages of 0.25, 0.5, 0.75 and 1, i.e. 125 test patches. The tables in the Appendix give the mean reflectance prediction error in terms of ΔE_{94} values, the maximal prediction error and the 95% quantile prediction error. In order to establish the "ground truth", we performed our experiments by first measuring the reflectance spectra of the complete set of 1072 two-by-two calibration tile patterns.

In order to build the training dataset, we need to select from the 1072 two-by-two tile patterns a subset whose reflectances should be measured. We are interested in the minimum number of measurements. The most important two-by-two calibration patterns are the 8 full-tone two-by-two colorant tiles, namely white, cyan, magenta, yellow, blue, green, red and black. These tile patterns are included in all selections of training patterns. The other training tile patterns are randomly selected from the two-by-two patterns set.

Table I (Appendix) demonstrates the performance of the proposed algorithm in predicting the reflectances of those two-by-two tile patterns which do not belong to the training set. These prediction accuracy tests were performed for 20 different random training sets. Three different training set sizes were chosen.

As shown in Table I, despite the fact that only a small fraction of two-by-two calibration patterns is measured, the two-by-two tile reflectance prediction accuracy is relatively high. In addition, the size of training set does not have a strong impact on the estimation accuracy.

4.2 Two-by-two model performance using estimated patterns

The two-by-two dot centering model aims at predicting the spectral reflectance of halftone prints. In order to verify the performance of our approach, after predicting the reflectances of a large majority of the two-by-two calibration tile patterns, we perform a comparison between the prediction accuracies of the two-by-two model where all two-by-two calibration tiles are measured and of the model incorporating measured and predicted two-by-two calibration tiles. We analyze the target halftones by calculating the number of repetitions of each calibration tile pattern within the color halftone print and predicting the color halftone reflectance spectrum using the Yule-Nielsen modified spectral Neugebauer (YNSN) Equation (14):

$$R(\lambda) = \left(\frac{\sum_{m=0}^{1071} i_m R_m(\lambda)^{1/n}}{\sum_{m=0}^{1071} i_m} \right)^n \tag{14}$$

where $R(\lambda)$ is the predicted color halftone reflection spectrum, i_m is the number of occurrences of each calibration two-by-two tile and R_m is the corresponding predicted or measured calibration two-by-two tile reflection spectrum. Table II (Appendix) shows the performance of the two-by-two dot centering model both for fully measured and for predicted two-by-two pattern reflectances. As expected, the prediction accuracy using estimated calibration tile pattern reflectance is lower. However, the accuracy difference is small. Therefore, significantly reducing the number of two-by-two pattern measurements does not significantly reduce the prediction accuracy of the two-by-two model.

For clustered-dot halftoning, the prediction accuracy difference between fully and partially measured calibration two-by-two tiles can be explained by the fact that in clustered-dot halftoning a large portion of two-by-two tile patterns are solid colorants. We performed the comparison of prediction accuracy also on blue noise diffused dither halftones where many different two-by-two tile patterns occur. Although the prediction accuracy is not as high as in clustered dot halftoning, it is still good and demonstrates the validity of our approach. For reference purposes, we also include the spectral prediction accuracy of the ink spreading enhanced Yule-Nielsen spectral Neugebauer model (IS-YNSN) which is calibrated with 44 calibration patches [7] printed with the same halftoning method and at the same orientation and screen frequencies as the test samples.

5. CONCLUSIONS

The two-by-two dot centering model predicts halftone spectral reflectances without knowledge of the specifications of the halftoning method and parameters. However, for a color halftone print it requires as input the reflectances of a large number of two-by-two calibration tile patterns. Spectral measurement of hundreds or thousands of tile patterns is cumbersome and time consuming. As an alternative, we learn with a small measured subset of the two-by-two calibration patterns a matrix giving the linear relationship between calibration pattern colorant surface coverages and the corresponding calibration pattern absorptance. This relationship is then used to predict the

absorptance and deduce the reflectance of all other two-by-two calibration patterns. The accuracy of the proposed method is verified by comparing the reflectance predictions relying on the two-by-two dot centering model based either on the measured full set or on the measured small subset of the two-by-two calibration tile patterns.

The mean color prediction error ΔE_{94} of the two-by-two model for a clustered-dot halftone at 100 lpi for a fully measured set of 1072 two-by-two calibration patterns is $\Delta E_{94} = 0.65$. When the two-by-two model is calibrated with only 72 two-by-two spectral measurements, the mean prediction error becomes $\Delta E_{94} = 0.74$. The accuracy of the proposed method is slightly lower for blue noise diffuse dither halftones where the fully measured calibration set yields a prediction error of $\Delta E_{94} = 0.71$ and the subset with the 72 measurements yields a prediction error of $\Delta E_{94} = 1.01$. Predicting a very large fraction of the two-by-two tile patterns enables using the two-by-two dot centering model for 3 or possibly 4-ink printing setups.

In the future, we intend to carry out further experiments with the goal of incorporating spatial tile pattern information into the matrix converting surface coverages to absorptances. We also intend to extend the prediction of two-by-two tile patterns to 4-ink halftone prints.

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APPENDIX

Table I. The color difference error for estimated two-by-two tile patterns.

Training set	Test set size	ΔE_{94}		
		Mean	95%	Max
8 colorants + 28 random samples	1036	2.05	4.46	9.53
8 colorants + 64 random samples	1000	1.97	4.57	8.33
8 colorants + 92 random samples	972	1.97	4.43	7.59

Table II. The prediction accuracy of the two-by-two dot centering model for predicting 125 clustered-dot and diffused dither halftones using (a) fully and (b) partially measured calibration two-by-two tile patterns.

Halftoning method	Training set size	ΔE_{94}			n-value	IS-YNSN mean ΔE_{94}
		Mean	95%	Max		
Clustered-dot 75 lpi	1072	0.87	1.57	2.39	2	0.43
	100	0.93	1.74	2.58	2	
	72	0.97	1.87	2.64	2	
	36	0.98	2.04	2.26	2	
Clustered-dot 100 lpi	1072	0.65	1.24	1.63	4	0.42
	100	0.78	1.47	2.10	3	
	72	0.79	1.66	2.35	3	
	36	0.76	1.46	2.10	4	
Clustered-dot 125 lpi	1072	0.61	1.07	2.19	7	0.57
	100	0.71	1.36	2.14	6	
	72	0.74	1.52	2.40	6	
	36	0.76	1.62	2.47	7	
Diffused dither	1072	0.71	1.44	1.74	14	1.08
	100	0.97	2.18	3.58	14	
	72	1.01	2.25	4.52	14	
	36	1.20	2.36	4.46	14	