Reducing auto moiré in discrete line juxtaposed halftoning

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ABSTRACT

Discrete line juxtaposed halftoning creates color halftones with discrete colorant lines of freely selectable rational thicknesses laid-out side by side. Screen elements are made of parallelogram screen tiles incorporating the discrete colorant lines. The repetition of discrete colorant lines from one screen element to the next may create auto moiré artifacts. By decomposing each supertile into screen element tiles having slightly different rational thicknesses, we ensure that successive discrete colorant lines have different phases in respect to the underlying pixel grid. The resulting repetition vector is different from one discrete line to the next discrete line of the same colorant. This strongly reduces the original auto moiré artifacts.

Keywords: Auto moiré, halftone screen, juxtaposed halftoning, discrete line, aliasing

1. INTRODUCTION

Juxtaposed halftoning is a special halftoning method where colorants formed by inks and ink superpositions are placed side by side. Juxtaposed halftoning is necessary when printing with special inks such as opaque or metallic inks. Recently, we introduced a new juxtaposed halftoning algorithm which creates side by side laid-out colorant halftone line screens without limiting the number of colorants [1]. *Discrete line juxtaposed halftoning* is based on the definition of a discrete line introduced by Reveillès [2, 3] which enables creating discrete lines at subpixel precision. The screen elements are formed by discrete line segments whose thicknesses are set according to the desired colorant surface coverages.

In classical color halftoning methods, moiré generally results from the superposition of two or more periodic superposed ink layers. A well-known solution to avoid the moiré produced by the superposition of layers is to rotate them in respect to each other by 30° [4]. Juxtaposed halftoning does not create superposition moiré, since ink layer dots are laid-out side by side. However there are aliasing effects occurring during line discretization on the output device grid. The repetition of these aliasing effects creates undesired low frequency components called auto moiré [5].

Auto moiré, also called internal moiré, results from the interference between the halftone and the device grid. In general, methods to reduce auto moiré aim at breaking regular patterns by randomization. They either try to vary the period of halftone dots [6, 7] or to correct quantization errors by an error diffusion process [8, 9]. It is also possible to suppress auto moiré artifacts with antialiasing filters [10, 11].

In this contribution, we try to characterize the auto moiré by analyzing the discrete layout of the digital line segments forming the halftone colorants. We try to break the single period screen element repetition vector into two different repetition vectors. Judicious choice of these repetition vectors reduces the visible auto moiré artifact.

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2. DISCRETE LINE GEOMETRY

In discrete line juxtaposed halftoning, both the screen element tiles and their colorant "dots" are formed by discrete line segments. Let us therefore introduce the basics of discrete line geometry. A set of points (x, y) in $\mathbb{Z}^2$ belongs to the discrete line $D(a, b, \gamma, w)$ if and only if each member of this set satisfies [2, 3]

$$\gamma \le ax - by < \gamma + w \tag{1}$$

where a and b are integer values which define the line's slope, $\gamma \in \mathbb{Z}$ indicates its position in the plane and $w \in \mathbb{Z}$ determines its thickness. Due to symmetry it is enough to consider the case where $|a| < |b|$, i.e. the line's absolute slope is smaller than 1. The arithmetic thickness parameter w controls the vertical thickness and the connectivity of the line. If $w < |b|$, the line is a disconnected *thin line*, if $w = |b|$, the line is called *naive digital line* and has exactly the vertical thickness of 1 and if $w > |b|$, the line has a vertical thickness greater than 1.

A discrete line with $0 < |a| < |b|$ has two *Euclidean* support lines. The superior support line is given by

$$y_{\text{sup}} = \frac{a}{b}x - \frac{\gamma}{b} \tag{2}$$

and the inferior support line is given by

$$y_{\text{inf}} = \frac{a}{b}x - \frac{\gamma + w}{b}. \tag{3}$$

Figure 1 shows a thick discrete line and its support lines.

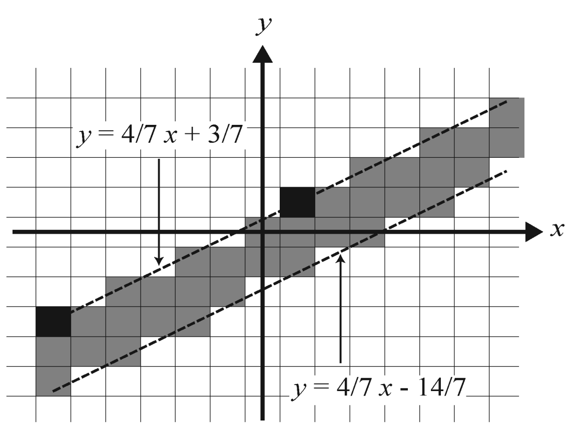

Figure 1. A thick discrete line with $a = 4$, $b = 7$, $\gamma = -3$ and $w = 17$. The vertical line thickness is $w/b = 17/7$. The "first pixels" colored in black are those where $ax - by = \gamma$, here for the left hand one $4\cdot(-6) - 7\cdot(-3) = -3$.

A given naive digital line segment repeats itself after b pixels in the horizontal direction. Figure 2 shows a discrete naive line whose periodicity has been highlighted.

3. AUTO MOIRE IN DISCRETE LINE HALFTONING

The screen element incorporating the discrete colorant line segments is formed by a parallelogram made of a discrete line segment having a given thickness and orientation. The parallelogram forming the screen element is defined by its sides given by vectors [0 T] and [b a] where T is the *vertical thickness* of the discrete line and a/b is the discrete line slope (Figure 3). A library is constructed with parallelogram screen elements of varying "dot" surface coverages comprising all possible rational discrete line thickness variations. As shown in Figure 3, halftoning a gray image is performed using these bilevel screen elements. The successive colorant "dots" are created by accessing and combining the binary screen elements stored within the library. The discrete line drawing algorithm as well as the grayscale and color halftoning methods are thoroughly described in [1].

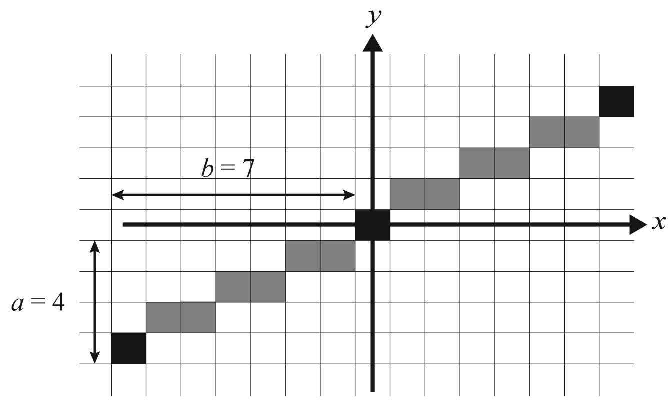

Figure 2. A discrete naive line with a slope of a/b repeats the same structure every b pixels (a = 4 and b = 7).

In discrete line halftoning the parallelogram screens made of discrete line segments repeat themselves in both directions within the discrete plane. To achieve an improved halftone appearance (less visible halftone) on a device with limited resolution, the frequency may be increased by decreasing the vertical period T. However, when the frequency is increased, auto moiré artifacts may appear. When the device resolution is high compared to the screen frequency, the auto moiré artifacts tend to vanish. These artifacts occur both in black and white and color halftones. For the sake of simplicity, we limit our analysis to simpler black and white colorant halftones. Solutions for black and white halftones also apply to color halftones.

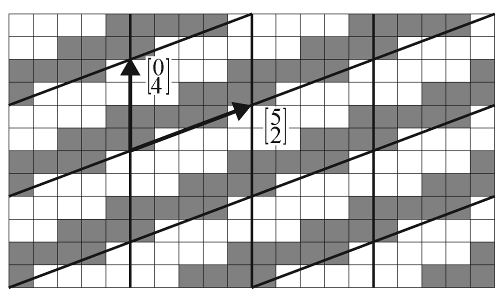

Figure 3. An output halftone image and the parallelogram screen element. The input image is a contone grayscale image with 45% surface coverage. The vertical thickness T is 4 and the slope is a/b = 2/5.

Figure 4 shows three discrete line halftones and their magnified bitmaps having identical line slope (a/b = 4/7) but different vertical periods T, namely 10, 8 and 6 respectively. To make the example clear, a colorant surface coverage was chosen at which the thickness of the discrete line is 8/7 and therefore has a thickness of two pixels at a certain location. When printing these bitmaps at 600 dpi, we obtain three patches of different appearances. While the patch with the lowest frequency (T = 10) exhibits no perceptually strong artifact, the two other patches show a vertically disturbing visual effect, at positions where the discrete line has a thickness of two pixels. On displays, anti-aliasing methods may avoid this artifact by adapting the gray levels of neighboring pixels [12]. However, on a binary printer this solution is not applicable.

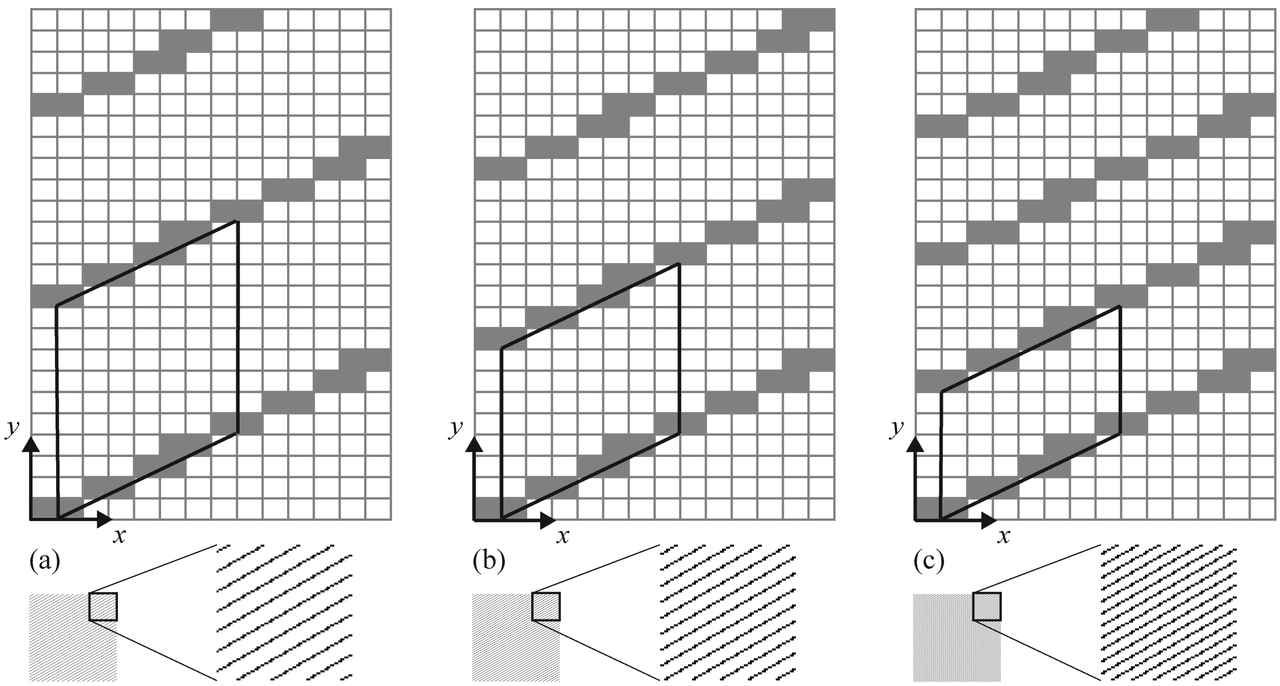

Figure 4. Three halftone patches and their corresponding bitmaps. The discrete colorant line slope is $a/b = 4/7$ and the vertical thickness for figures 4a, 4b and 4c is 10, 8 and 6 respectively. The chosen colorant surface coverage for each patch corresponds to a discrete line with thickness $w/b = 1 + 1/b$, i.e. at a certain location the discrete line is two pixels thick. The bottom row shows the halftone patches at the real size at 600 dpi and also enlarged by a factor of 5×5.

4. SUPPRESSING AUTO MOIRE BY RATIONAL SUB-PERIODS

As Figure 4 shows, the vertical repetition of identical discrete line segments yields the auto moiré. Let us analyze the effect obtained by a rational repetition of discrete colorant lines. By displacing the discrete lines by a rational value, we are able to choose different locations of the plane where the two pixel height staircase occurs. According to the naive line drawing algorithm [1, 2], the pixels are drawn by rounding down the y coordinate of their superior support line given by Equation (2). The corresponding *remainder function* $r(x)$ of the discrete line is defined by

$$r(x) = \left\{ \frac{ax - \gamma}{b} \right\} \tag{4}$$

where the curly bracket denotes the Euclidean remainder. Remainder function $r(x)$ determines the vertical distance $r(x)/b$ between the superior support line and the discrete pixel center located just below it. The remainder function values include the set of integers between 0 and $b - 1$. The sequence of remainders characterizes the succession of segments forming the digital line and only depends on a and b. The coefficient γ determines the order of this sequence [3]. Note that a staircase pixel occurs each time the difference between successive remainders becomes negative, i.e. $r(x_{i+1}) - r(x_i) < 0$.

A discrete line has a vertical thickness that is a multiple of $1/b$. Therefore, one may replicate the discrete line segment at a vertical offset t/b, where $t \in \mathbb{Z}$. The remainder function of the replicated line is

$$r_2(x) = \left\{ \frac{ax - (\gamma + t)}{b} \right\} = \left\{ \frac{ax - \gamma}{b} - \frac{t}{b} \right\}. \tag{5}$$

Using the following identity

$$\left\{\frac{\delta+\upsilon}{\varepsilon}\right\}=\left\{\frac{\delta}{\varepsilon}\right\}+\left\{\frac{\upsilon}{\varepsilon}\right\}-\left[\frac{\left\{\frac{\delta}{\varepsilon}\right\}+\left\{\frac{\upsilon}{\varepsilon}\right\}}{\varepsilon}\right]\varepsilon \tag{6}$$

where the square bracket denotes the quotient of the Euclidean division, we obtain

$$r_2(x)=\left\{\frac{ax-\gamma}{b}\right\}+\left\{\frac{-t}{b}\right\}-\left[\frac{\left\{\frac{ax-\gamma}{b}\right\}+\left\{\frac{-t}{b}\right\}}{b}\right]b. \tag{7}$$

When t is a multiple of b, $\left\{\frac{-t}{b}\right\}=0$ and the remainder function $r_2(x)$ is the same for the original and the replicated discrete line. If t is not a multiple of b, the remainder function differs from the original one according to the value of t. Consequently, the discretization of the original line and of the replicated line will be shifted in respect to each other.

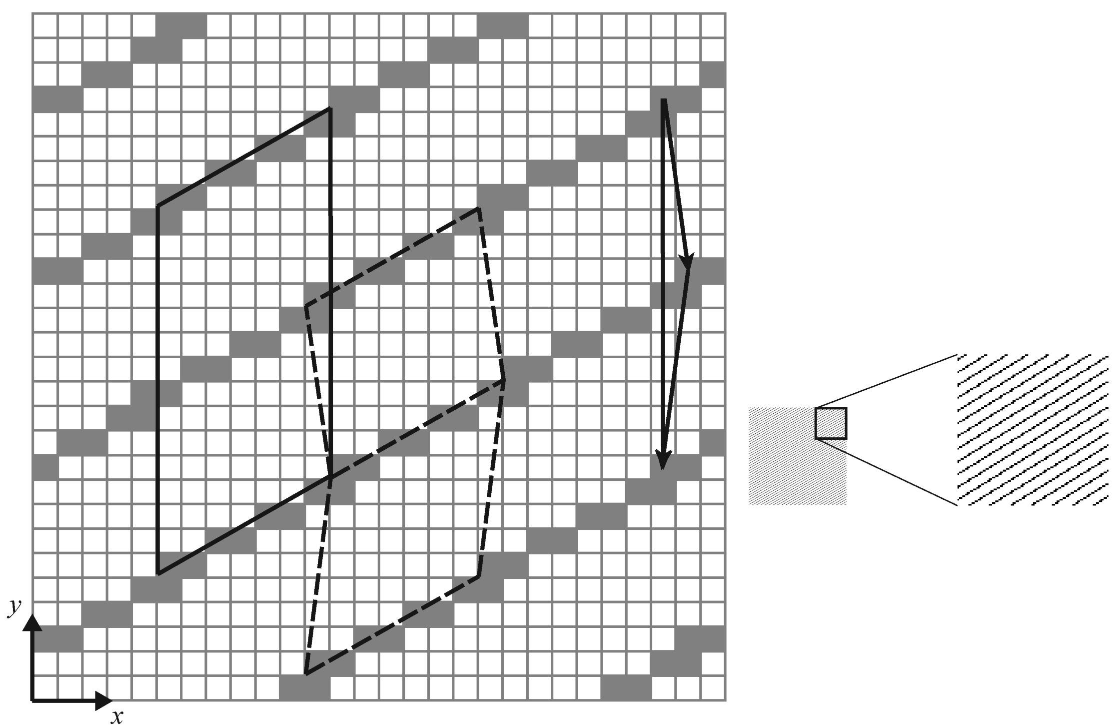

Figure 5. Staircase repetition vectors as well as supertile (solid line) and screen element tiles (dashed line) of the halftone with a/b = 4/7, T_1 = 52/7 and T_2 = 53/7. The right-hand view shows the halftone patches at the real size at 600 dpi and also enlarged by a factor of 5×5.

When replicating the discrete lines with an integer vertical offset, i.e. when t is a multiple of b, each staircase is repeated at a certain vertical distance. In case of rational repetition of discrete lines, the staircase is replicated in a direction different from the vertical. We look for successive *staircase repetition vectors* that break the visually objectionable base frequency induced by a single repetition vector.

Staircase repetition vectors define the relative position of corresponding staircases in successive colorant lines. To compute the staircase repetition vectors, the coordinates of first pixels of the successive lines are computed. The vector pointing from the first pixel of the original discrete line to the corresponding first pixel of the next discrete line is the repetition vector. We define the first pixel of the discrete line D (a, b, γ, w) as a pixel whose center $\mathbf{p}_1(x_1, y_1)$ intersects the superior support line (black pixels in Figure 1). By definition, the remainder function given by Equation (4) at the integer x coordinate of this pixel has a value of zero. In other words

$$ax - \gamma = k \cdot b \qquad (k \in \mathbb{Z}). \tag{8}$$

The first pixel of the original discrete line has according to Equation (2) the integer ordinate

$$y_1 = \frac{ax_1 - \gamma}{b}. \tag{9}$$

We choose a new value of γ for the replicated discrete line. Then we compute its first pixel coordinate $\mathbf{p}_2(x_2, y_2)$ with the same procedure as for the original line. Having computed the first pixels of two successive discrete lines, the first to second discrete line repetition vector is computed by subtracting the two coordinates, i.e. $\mathbf{v}_{12} = \mathbf{p}_2 - \mathbf{p}_1$.

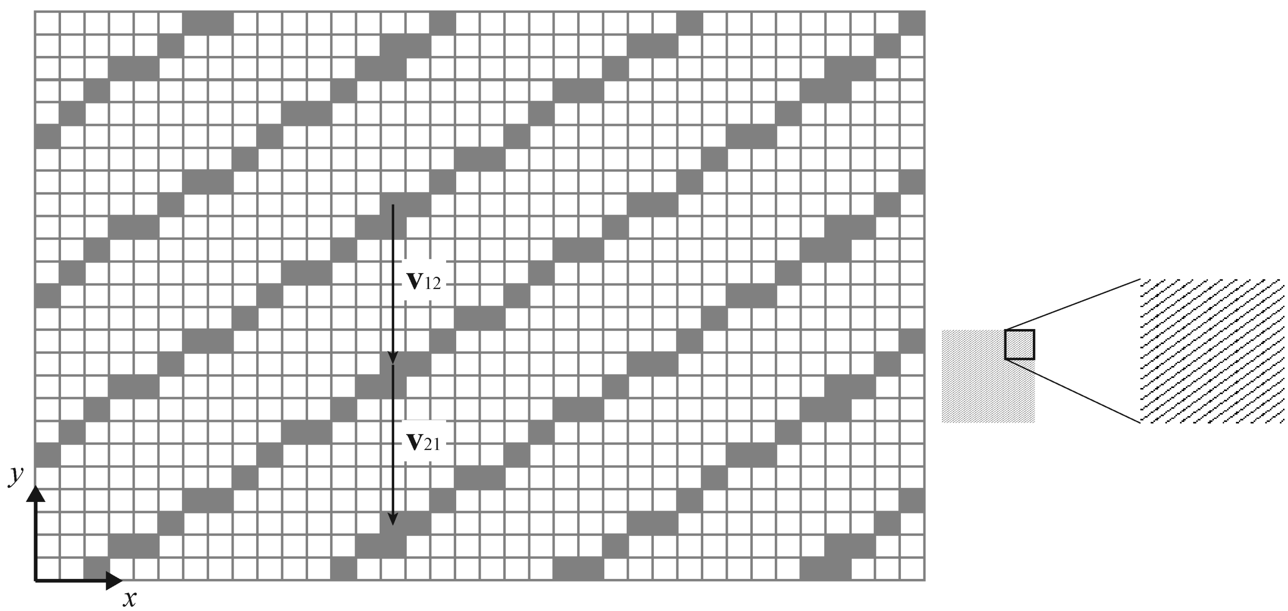

Figure 6. Auto moiré in the vertical direction for the halftone with line slope a/b = 13/18 and T = 7. The right-hand view shows the halftone patches at the real size at 600 dpi and also enlarged by a factor of 5×5.

Since there is a need for paving the plane with a discrete supertile of screen elements, we need for that supertile an integer displacement which is the result of the addition of several rational screen element displacements. For the sake of simplicity, we are particularly interested in a small number of rational displacements, for example an integer displacement divided into two rational displacements. Let us, consider a discrete supertile parallelogram with a slope of 4/7 and a vertical period T = 15. One may divide the supertile vertical thickness T = 15 into two rational thicknesses, for example T_1 = 52/7 and T_2 = 53/7. This results in two screen element tiles of slightly different vertical thicknesses. As shown in Figure 5, because of the rational vertical displacements between one screen element tile and the next, the second discrete colorant line is horizontally shifted in respect to the first one.

Since the sum of the two rational displacements yields an integer displacement, the third discrete colorant line has exactly the same staircase layout as the first discrete colorant line at a vertical distance $T = 15$ pixels from it. While the actual screen frequency of the halftone is close to those shown in Figure 4b and 4c, the visible auto moiré is considerably reduced (see Figure 5).

Replicating discrete lines with two arbitrary selected vertical rational displacements does not automatically solve the problem of auto moiré. As an example, consider the halftone with a line slope composed of large a and b, $a/b = 13/18$. Figure 6 shows the magnified pixel map as well as the patch in real size for an integer vertical thickness $T = 7$. It exhibits a clearly visible auto moiré artifact in form of vertical lines. One may try to bisect the vertical period $T = 15$ to obtain a screen element vertical period almost equal to 7 (15/2), by using two rational periods $T_1 = T_2 = 135/18$. The resulting halftone patch still incorporates the same auto moiré intensity but at an orientation different from the vertical orientation. Due to the two identical rational vertical displacements T_1 and T_2, the displacement vector $\mathbf{v}_{21}$ repeats itself between successive discrete lines. The orientation of this repetition vector defines the orientation of the auto moiré in the halftone patches (Figure 7).

In order to avoid the auto moiré in a new orientation, the challenge resides in finding the optimum pair of staircase displacement vectors. The only parameter that can be changed to obtain different staircase repetition vectors is the rational distance between the original discrete colorant line and its replicated instance. For the latter case of $a/b = 13/18$, by choosing two different repetition vectors, such as $T_1 = 134/18$ and $T_2 = 136/18$ the artifacts are significantly reduced but not completely eliminated (Figure 8). When b (and consequently a) is a large number, one period of a discrete line is composed of several distinct segments. Hence, even when avoiding identical repetition vectors, some new dot configurations may come up and produce a visible auto moiré. With small slope parameters, there are potentially less moiré generating configurations.

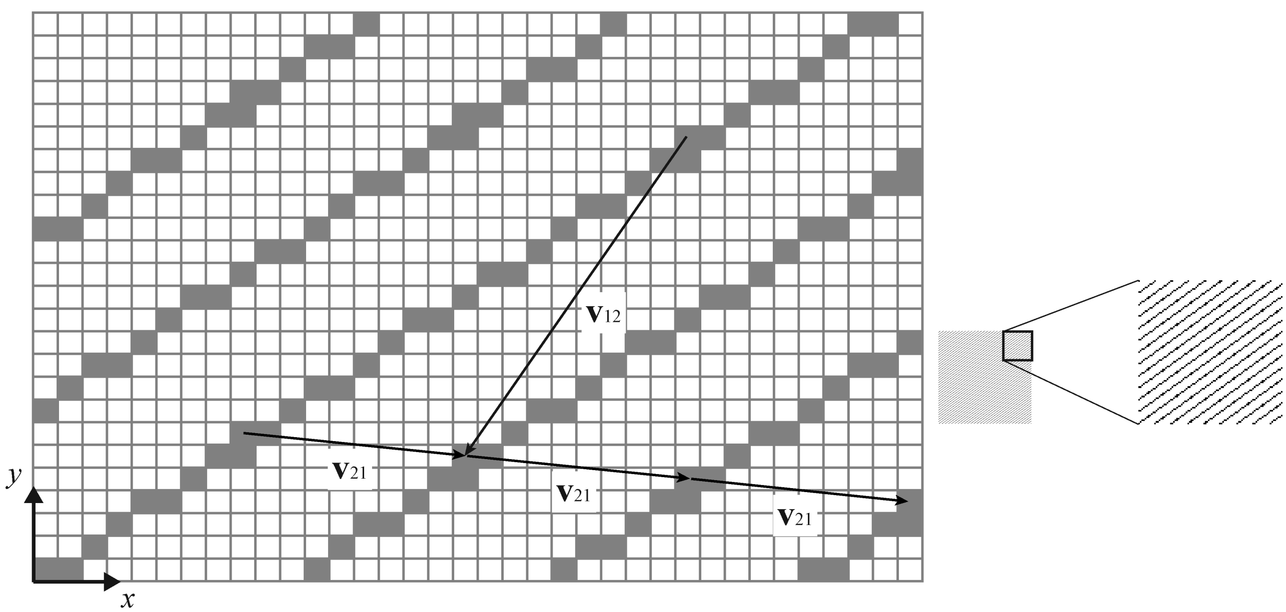

Figure 7. Auto moiré in new direction for the halftone with line slope $a/b = 13/18$ and two rational period $T_1 = T_2 = 135/18$. The auto moiré orientation is shown by the repetition vector $\mathbf{v}_{21} = (9, -1)$. The right-hand view shows the halftone patches at the real size at 600 dpi and also enlarged by a factor of 5×5.

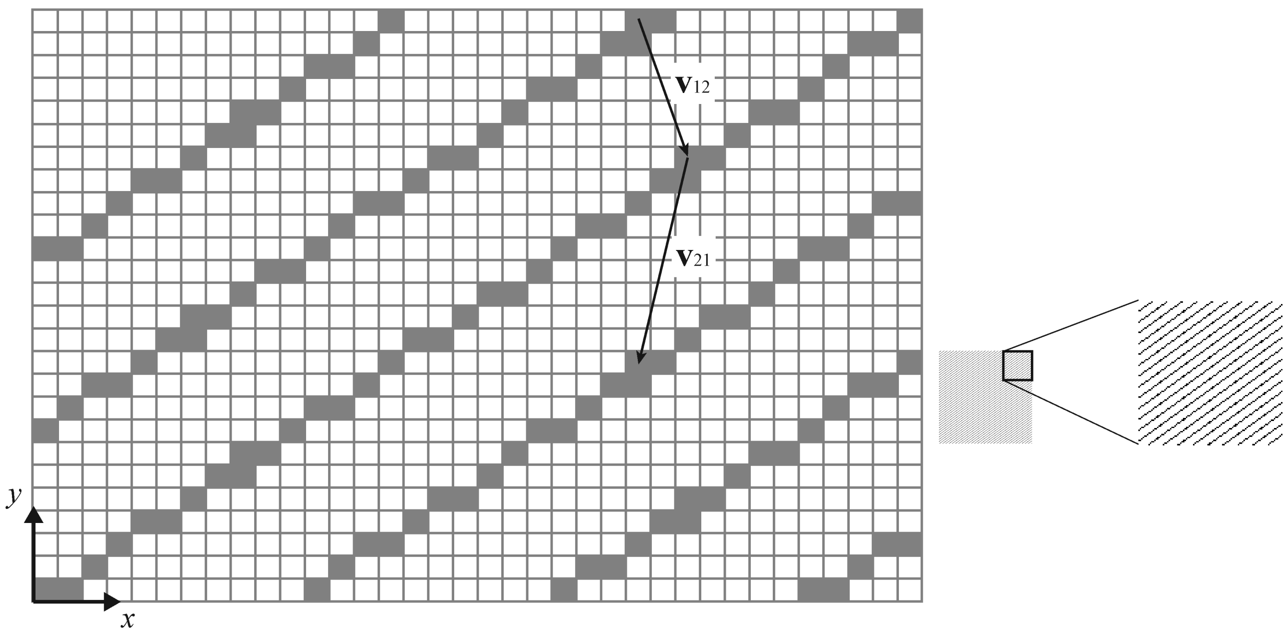

Figure 8. Halftone with line slope a/b = 13/18 and two rational period T_1 = 134/18 and T_2 = 136/18. The repetition vectors are $\mathbf{v}_{12}$ = (2, –6) and $\mathbf{v}_{21}$ = (–2, –9). The right-hand view shows the halftone patches at the real size at 600 dpi and also enlarged by a factor of 5×5.

5. CONCLUSIONS

Auto moiré is due to the periodic occurrence of staircases indicating that the discrete line becomes thicker. It occurs when paving the plane with discrete parallelogram screen elements. In order to reduce these auto moiré artifacts, we divide a supertile into two screen element tiles. By selecting different rational distances between successive discrete lines of the same colorant, one ensures that they have different phases in respect to the device grid. For two screen elements within one supertile, we select a repetition vector that is different from one discrete line to the next discrete line of the same colorant. This breaks the monodirectional auto moiré artifact into two less visible artifacts having different orientations. For specific screen element periods and orientations we have been able to reduce the auto moiré artifact. By finding an appropriate optimization criterion, we intend to generalize this approach to a larger set of screen element periods and orientations.

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