

SI 2: Curvilinear 1D moiré mathematical model

If (x, y) are defined as the original space coordinates and (x_t, y_t) as the destination space coordinates, it is possible to relate them by the two continuous functions representing the geometric transformation of the base layer: $x=h_x(x_t, y_t)$ and $y=h_y(x_t, y_t)$. Then, if we define the continuous function $y=g_y(x_t, y_t)$ as the revealing layer mapping from the destination to the original space coordinates, the resulting moiré in the destination space is described¹² by the geometric transformations $x=m_x(x_t, y_t)$ and $y=m_y(x_t, y_t)$, given by :

$$\begin{aligned} x &= m_x(x_t, y_t) = h_x(x_t, y_t) + (h_y(x_t, y_t) - g_y(x_t, y_t)) \cdot \frac{t_x}{T_r - t_y} & ; \\ y &= m_y(x_t, y_t) = h_y(x_t, y_t) \cdot \frac{T_r}{T_r - t_y} - g_y(x_t, y_t) \cdot \frac{t_y}{T_r - t_y} \end{aligned} \quad (3)$$

where T_r is the revealing layer grating period in the original space and $\vec{t} = (t_x, t_y)$ is the base band replication vector in the original space (see Fig. 1). Note that in the original space, the revealing layer is formed by a grating of horizontal sampling lines (transparent thin lines or lenticular lenses parallel to the x-axis). The set of equations (3) enables the selection of the layout of two layers to obtain the layout of the third layer, for the set comprising the base, the revealing and the moiré layers. By solving Eqs. (2) for the base layer transformation (h_x, h_y) , we obtain the layout of the base layer as a function of the layouts of the moiré and of the revealing layer:

$$\begin{aligned} h_x(x_t, y_t) &= (g_y(x_t, y_t) - m_y(x_t, y_t)) \cdot \frac{t_x}{T_r} + m_x(x_t, y_t) \\ h_y(x_t, y_t) &= g_y(x_t, y_t) \cdot \frac{t_y}{T_r} + m_y(x_t, y_t) \cdot \frac{T_r - t_y}{T_r} \end{aligned} \quad (4)$$

In order to create a circular moiré layout, in which the individual moiré shapes move inwards or outwards along radial or spiral trajectories, a circularly transformed moiré layer revealed by a rectilinear revealing layer grating (identity transformation: $g_y(x_t, y_t) = y_t$) is required. Considering the circular moiré transformation given by

$$\begin{aligned} m_x(x_t, y_t) &= \frac{\pi - a \tan(y_t - c_y, x_t - c_x)}{2\pi} w_x \\ m_y(x_t, y_t) &= f\left(\sqrt{(x_t - c_x)^2 + (y_t - c_y)^2}\right) \end{aligned} \quad (5)$$

where (c_x, c_y) specifies the centre of the circular moiré layout, w_x is the width of the moiré in the original space and $f(\rho)$ is a vertical scaling function enabling the height to length ratio of the moiré tile to be kept constant when the moiré moves from the interior to the exterior of the circular layout.

The layout of the base layer capable of creating the circular moiré is given by transformation (h_x, h_y) which is obtained by inserting Eqs. (5) into Eqs. (4) and with $g_y(x_t, y_t) = y_t$:

$$\begin{aligned} h_x(x_t, y_t) &= (y_t - f(\sqrt{(x_t - c_x)^2 + (y_t - c_y)^2})) \frac{t_x}{T_r} + \frac{\pi - a \tan(y_t - c_y, x_t - c_x)}{2\pi} w_x \\ h_y(x_t, y_t) &= y_t \frac{t_y}{T_r} + f(\sqrt{(x_t - c_x)^2 + (y_t - c_y)^2}) \frac{T_r - t_y}{T_r} \end{aligned} \quad (6)$$

Figure 7(a) shows an example of a rectilinear base layer superimposed with a horizontal revealing layer (black horizontal lines) and the resulting moiré image. Figure 7(b) represents the resulting transformed circular base layer and resulting moiré, according to equations 5 and 6 with $f(x) = x$. In the circular moiré Figure (7b)

the radial position corresponds to the vertical axis (y) and the angular position corresponds to the horizontal axis (x) of the rectilinear moiré shown in Figure 7(a).

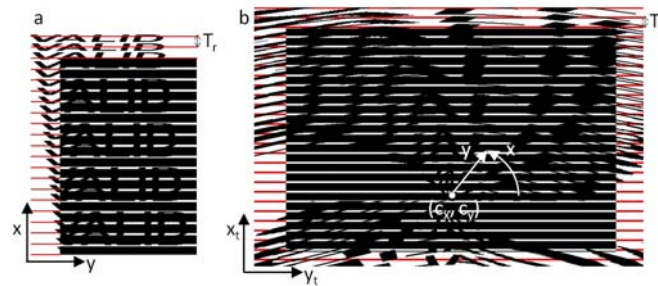


Figure 7: (a) Example of a rectilinear base band grating made of a replicated, vertically compressed text (“VALID”), with the moiré revealed by a transparent line grating (revealing layer) and (b) the geometrically transformed base layer and the corresponding moirés using the same revealing layer. In the transformed base layer, x - and y -axis from the original space correspond to the angular and radial coordinates, respectively.